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Distinct Pathways From Parental Beliefs and Practices to Children's Numeric Skills

Marina Vasilyeva[a,b], Elida Laski[a], Aleksandr Veraksa[b], Lindsey Weber[a], and Daria Bukhalenkova[b]

[a]Boston College, Chestnut Hill, MA, USA; [b]Lomonosov Moscow State University, Moscow, Russia

ABSTRACT
It is imperative to identify contextual factors contributing to the development of early math skills, considering their role in later academic achievement. To pursue this goal, the present study investigated the paths connecting parental beliefs and practices during the preschool years to children's numeric skills at the end of kindergarten (N = 98). Results were consistent with theoretical predictions of specific relations between particular types of parental input and different aspects of number knowledge. Direct math learning activities mediated the relation between parental beliefs and children's *number identification skills*. Daily activities involving quantitative components mediated the relation between parental beliefs and children's *numerical magnitude understanding*. Both types of activities predicted *arithmetic skills* that integrate the basic aspects of symbolic number knowledge. These findings contribute to developmental theory by specifying how characteristics of children's environments are related to particular aspects of their development, which is critical for informing intervention work to improve early math skills.

Early math knowledge is one of the strongest predictors of subsequent academic achievement (Duncan et al., 2007; National Mathematics Advisory Panel, 2008). A key aspect of this knowledge is the mastery of symbolic number skills, which provide a foundation for learning formal mathematics. At the earliest stages of schooling, children exhibit substantial differences in symbolic number skills. To better understand the factors associated with numeric development prior to school entry, the present study investigated the relationship between activities in which parents engaged children during the last year of preschool and the symbolic number skills of these children in kindergarten.

The current research was situated within the constructivist framework (Vygotsky, 1978) positing that social processes play a significant role in the growth of intellectual skills. A further theoretical analysis (Bronfenbrenner, 1979) has delineated two kinds of social processes that influence development: proximal processes, which refer to direct interactions between the child and the environment, and distal processes that do not involve the child directly but may have indirect effects. A key proximal mechanism through which cultural knowledge is transmitted to young children is the interaction between the parent and the child. Variability in the amount and nature of such interactions predicts variability

CONTACT Marina Vasilyeva ✉ vasilyev@bc.edu Lynch School of Education, Boston College, 140 Commonwealth Ave., Chestnut Hill, MA 02467, USA.

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in a wide range of child outcomes (Huttenlocher, Waterfall, Vasilyeva, Vevea, & Hedges, 2010; LeFevre et al., 2009; Manolitsis, Georgiou, & Tziraki, 2013). This proximal process does not operate in isolation, however; differences in input have been linked to distal factors, such as parents' beliefs. For example, parents' views on child development and learning have been identified as a source of variability in their interactions with children (Kim, Murdock, & Choi, 2005; Stipek, Milburn, Clements, & Daniels, 1992).

Given the links between proximal and distal factors, it is important to investigate them simultaneously to develop a better understanding of the social processes facilitating the growth of child knowledge. Theoretically, this approach allows researchers to develop a more cohesive model of the pathways through which the home environment provides a context for child development. Practically, it may lead to designing parent intervention programs that would not only inform parents about effective home activities for preschoolers, but also target the beliefs that may shape parents' motivation for engaging in such activities.

The present study investigated proximal and distal factors associated with math learning by examining the relations among a) children's symbolic number skills, b) parent–child math activities, and c) parents' beliefs about math and early learning. To distinguish these specific relations from more general associations between parents' input and children's skills, we controlled for overall parental enrichment activities and child intelligence.

Development of symbolic number skills

Acquiring symbolic numeric knowledge typically begins during preschool, as children learn number words and master counting principles (Geary, 2006; Opfer & Siegler, 2012). Later, children build on this foundation to acquire knowledge of the written number system. First, they establish associations between written numerals and corresponding number words (e.g., "5" = "five"). In addition to identifying numerals, children learn the relation between the symbol and the quantity it represents. This understanding leads to the ability to reason about numeric relations, such as making judgments about which number is "bigger" (i.e., corresponds to a larger magnitude). As children master these basic components of symbolic knowledge—number identification and numerical magnitude understanding—they develop more complex skills that integrate these components and allow for exact numerical computations.

Although the acquisition of symbolic number knowledge is a focus of math curricula in elementary school, there are experiences that support the growth of symbolic numeric skills even before school entry. Some number learning occurs through incidental exposure as children interact with their environment (Mix, Prather, Smith, & Stockton, 2014). Other symbolic number learning outside of school is mediated by spontaneous or deliberate interactions with more skilled social partners. Parents of preschoolers serve the role of such social partners by engaging children in math-relevant activities at home.

Relationship between home activities and children's math skills

Previous investigations have identified a variety of home activities that involve numerical components, such as counting objects, playing board games, and measuring. These investigations have utilized a range of methodologies, including parental surveys

(Blevins-Knabe & Musun-Miller, 1996; Huntsinger, Jose, & Luo, 2016; LeFevre et al., 2009; Saxe, Guberman, & Gearhart, 1987; Skwarchuk, Sowinski, & LeFevre, 2014), direct observations of parent–child interactions (Gunderson & Levine, 2011; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010), and experimental design examining the effects of parental input on children's math skills (Berkowitz et al., 2015).

The choice of a particular approach depends on the nature of the research questions. Experimental investigations allow researchers to explore causality, yet they are best suited for study of a narrowly defined activity. Direct observations provide a broader sampling of home input but are limited to activities occurring during the observation period. Parental reports, on the other hand, although less rigorous than experimental studies and direct observations, provide the most comprehensive view of the kinds of activities that occur in naturalistic contexts. The present study utilized parental reports and thus maximized the range of activities examined to investigate specific relations between particular activities and child outcomes.

Previous studies using parental reports have shown that math-related home practices varied in the extent to which they were decontextualized versus embedded in daily activities. Accordingly, math practices were categorized as formal or informal (LeFevre et al., 2009). This categorization was intended to capture the distinction between school-type learning that directly focused on math and daily interactions that incidentally involved numeric components. Yet subsequent studies have varied substantially in which activities were assigned to each category. Such variability was likely the result of classifying activities based on factor analysis of their frequencies within a given study, which meant that the same activity might be assigned to different categories in two different studies. For example, one study classified helping the child to measure ingredients as a formal activity because of its factor loading, whereas most other studies classified it as informal.

This inconsistency could be minimized if the classification of activities was based on the conceptual distinction between formal and informal categories, rather than factor analysis. The present study, thus, operationalized *formal math activities* a priori as those in which learning occurs as an abstract decontextualized exercise, where improving math skills is the focal goal of the activity. In contrast, *informal math activities* were operationalized as parental practices in which math is embedded in the context of a daily task and is peripheral to the main goal of that task. This framework allowed us to make theoretical predictions about relations between each type of activity and the development of symbolic numeric skills, as detailed in the "Present Study" section.

Relationship among parental beliefs, home activities, and children's math skills

Whereas home activities directly involve the child, parental beliefs represent distal processes that may relate to child outcomes indirectly via home activities. Previous studies have shown that parents' interactions with children are influenced both by their beliefs about themselves, such as self-efficacy, and their general views about child development, such as what skills should be acquired prior to school (Hatcher, Nuner, & Paulsel, 2012; Kim et al., 2005). These same types of beliefs have been shown to predict cognitive and behavioral skills in children (Englund, Luckner, Whaley, & Egeland, 2004; Sigel, McGillicuddy-DeLisi, & Goodnow, 2014). Taken together, the findings suggest that parent–child activities may mediate the relationship between parental beliefs and child outcomes. Indeed, a recent study provided evidence of this mediated relation in the context of literacy skill development in preschool children (Vasilyeva, Dearing, Ivanova, Shen, & Kardanova, 2017).

When considering the growth of math skills, several types of beliefs have appeared to be particularly relevant to parents' willingness to engage children in math-related activities. One of these beliefs is parents' *belief in the importance of academic preparation* of preschoolers. In a recent study, this type of belief was correlated with the frequency of formal math practices at home, which in turn predicted child performance on a math assessment (Skwarchuk et al., 2014). Another belief shown to influence parents' motivation to engage children in math activities pertains to the *perceived malleability of math skills*: Parents who view math ability as relatively fixed are less likely to engage children in math practice at home (Muenks, Miele, Ramani, Stapleton, & Rowe, 2015). But even if parents hold a malleable view of intelligence and are concerned about the child's school preparation, they may not focus on math either because they do not perceive it to be as important as, for example, literacy or because they are not confident in their own math skills. Thus, parents' beliefs about the *value of math knowledge* as well as their own *math self-efficacy* may affect the likelihood of their engagement in math activities.

Present study

This study investigated children's symbolic numeric skills as a function of parental beliefs and parent–child home math activities. Parental data were obtained through questionnaires administered when children were in the last year of preschool. Child data were collected through individual assessments a year later at the end of kindergarten. The study was part of a larger project on math learning conducted in Russia. The majority of children in Russia attend state-funded preschools regulated by federal curricular standards (Ministry of Education and Science of the Russian Federation, 2014). Thus, although preschool environments vary in quality due to differences among teachers, this variability is likely smaller than that in the United States where preschools vary both in terms of teachers and curricular approaches. In a context where curricular differences among preschools are minimal, differences in the home environment may be particularly consequential in accounting for variability in children's early academic skills.

In examining relations between home math activities and symbolic number knowledge, we built on previous work that showed parent–child activities to be predictive of children's math performance (e.g., Huntsinger et al., 2016; Skwarchuk et al., 2014). Yet in this previous work, a single summary score was used to encompass a broad range of math skills. A theoretical analysis, however, suggests that this approach may have obscured nuances in the relationship between types of activities and math knowledge. Consider the two basic aspects of symbolic number knowledge: numeral identification and numerical magnitude understanding. Whereas systematic practice with reading or writing numerals might facilitate number identification, it is unlikely to be sufficient for establishing strong numerical magnitude associations. In contrast, the latter may benefit from contextualized activities that involve linking numbers to quantity, such as when children help set a dinner table with a certain number of place settings.

This analysis suggests that specific types of home activities differentially predict specific aspects of symbolic numeric skills. Consequently, the present study included separate measures for different components of symbolic number knowledge: numeral identification, magnitude understanding, and arithmetic. Further, based on the theoretical analysis, we made three predictions: 1) The frequency of formal math activities would predict number identification skills; 2) the frequency of informal math-related activities would predict numerical magnitude understanding; and 3) arithmetic skills, which are based on both symbol knowledge and numerical magnitude understanding, would be predicted by both formal and informal activities.

Home math activities

To measure the frequency with which parents engaged children in various math activities, we generated a parental survey. Prior to generating the survey, we conducted interviews with a separate group of preschool parents to get a better sense of home math practices in Russia. This pilot work indicated that Russian parents engaged in activities similar to those documented in research conducted in the United States and Canada; thus, this information allowed us to include established survey items in the present study. Further, parents' descriptions of math-related interactions, which often included contextual information, were useful for the preliminary testing of our predictions. That is, they allowed us to see whether math learning was focal or peripheral to the main goal of a given activity and elucidated the math skills that were involved in that activity.

Most activities that parents described as isolated math exercises appeared to target the "technical" knowledge of number symbols or rote counting (e.g., writing numerals or reciting numbers). In contrast, math-related interactions occurring in the context of daily activities appeared likely to facilitate the mapping between numbers and corresponding magnitudes—either because they provided perceptual referents for numbers (e.g., using three apples for a recipe) or because they encouraged thinking about magnitude (e.g., when buying a toy, $20 is more than $5). Thus, the pilot data provided preliminary support for our theoretical analysis and suggested that different kinds of activities are associated with different aspects of symbolic number knowledge.

Parental beliefs

Based on previous findings and our own analysis of which beliefs might affect parents' math interactions, we assessed four types of beliefs: views on the importance of academic preparation, beliefs about the malleability of math skills, perceived value of math, and math self-efficacy. Several studies examined subsets of these beliefs in relation to home activities without considering child outcomes (e.g., Hatcher et al., 2012). A few studies that did include child measures used standardized assessments rather than testing specific math skills (e.g., LeFevre et al., 2009). The present study extended this work by examining parental beliefs in relation to different types of math activities and specific numeric skills in children. We hypothesized that each type of belief may have a unique contribution to the frequency of different types of math activities. Although we did not have strong predictions about the specificity of this relation, we considered the possibility that math value and self-efficacy may play a greater role than other beliefs in predicting the frequency of informal activities. These two types of beliefs may capture a positive attitude toward mathematics that combines interest and confidence in the domain. Parents who have this attitude may be particularly likely to pay attention and to draw their child's attention to math-related aspects of everyday situations.

In sum, the present study examined the specificity of the relationship among parental beliefs, home math activities, and the components of symbolic numeric skills. For each skill, we tested a model that included all potential distal (beliefs) and proximal (activities) predictors to identify significant mediation paths leading to specific outcomes. To increase the specificity of the model, we controlled for parents' general enrichment activities and for children's general intellectual skills.

Method

Participants

Participants were recruited through municipal preschools in Moscow, Russia. Initially, 121 families were contacted; 12 of them did not return the questionnaires. Of the remaining families, 11 children were not present at the time of kindergarten testing. Thus, the analytic sample included 98 children and their parents. At the time of testing, the participating children (52% girls) were aged an average of 6;10 (*SD* = 4.8 months). In the Russian educational system, children enter kindergarten 1 year later than in the United States, and kindergarten classrooms are part of the early education system that includes preschools. All children in this study were exposed to the same preschool and kindergarten curriculum. The participating parents identified themselves as the primary caregivers; most were mothers (93%). The parents reported the following levels of education: 13% high school diploma, 27% vocational certificate, 5% incomplete college, 53% college degree, 2% graduate degree.

Measures

Parental questionnaire

The questionnaires, distributed at the end of preschool, contained a wide range of items. In the present study, we used data from two sections. One section assessed the frequency of parent–child activities using a Likert scale: For each activity, the parent had to select a frequency category from 1 = "very rarely" to 6 = "more than once daily." This section included 18 items, comprising three subscales (see Appendix A for a full list of items). Six items described activities with a direct focus on math learning, such as teaching children how to write numerals. Six other items described contextualized daily activities with embedded math components, such as talking about prices when shopping. These 12 math items were largely derived from measures used in previous research on parent–child home activities (e.g., Huntsinger, Jose, Liaw, & Ching, 1997; LeFevre, Polyzoi, Skwarchuk, Fast, & Sowinski, 2010; LeFevre et al., 2009). Although, as mentioned earlier, individual math activities were not always classified in the same way from one study to another, there was a convergence across the majority of studies in the classification of activities. Our current categorization of items as either formal or informal math activities was consistent with the most common way each activity was classified in previous work. In addition, to control for quality of parenting, 6 items assessed general enrichment activities, such as going to the library or doing arts and crafts. These items were used in our previous research (Vasilyeva et al., 2017). All three subscales showed good internal consistency: Cronbach's alphas were .91 for Formal Math Activities, .82 for Informal Math Activities, and .78 for General Enrichment Activities. Three separate scores were computed to represent the mean frequency of activities for each subscale.

The second section of the questionnaire assessed parental beliefs. For each statement representing a particular belief, the parent had to select a response indicating the degree of agreement with that belief. Responses ranged from 1 = "strongly disagree" to 6 = "strongly agree." The section included four subscales (with six items in each) representing different types of beliefs. The first scale—*Importance of School Preparation*—assessed parents' views on the importance for children to develop certain math and literacy skills to be considered ready for school. Each item began with the same statement ("The child who is ready for school should be able to …"), followed by a skill description (e.g., "solve simple math problems" or "recognize letters"). Psychometric properties of this scale were examined with a principal component analysis (PCA) of standardized residuals (Linacre, 2011). The PCA generated standardized residual variance values close to 1 (0.9–1.3) in eigenvalue units, which was interpreted as evidence of unidimensionality (Smith, 2002). This finding indicates that views on the importance of school preparation are likely to reflect a general, rather than a domain-specific, attitude.

The other three scales assessed math-specific beliefs that could have affected parents' motivation to engage children in math activities: value, self-efficacy, and growth mindset (ability vs. effort). The scales were based on similar measures used in previous research on motivation (Eccles & Wigfield, 1995; Fennema & Sherman, 1976; Leslie, Cimpian, Meyer, & Freeland, 2015). The *Math Value* scale assessed the perceived value of mathematical knowledge (e.g., "Math is important in everyday life" or "I consider math to be one of the most important subjects in school"). The *Math Self-Efficacy* scale assessed parents' perceptions of their math skills and their willingness to engage in math tasks (e.g., "It was easy for me to get good grades in math" or "I like solving challenging math problems"). The *Math Ability Versus Effort* scale evaluated beliefs about the nature of success in math, while pitting the role of effort against raw ability (e.g., "With the right amount of effort and dedication, anyone can become successful in math" or "Success in math requires a special ability that can't be taught"). Responses on this scale were coded such that higher scores indicated a greater belief in the role of effort in math success.

All four parental belief scales showed appropriate levels of internal consistency: Cronbach's alphas were .84 for Importance of School Preparation, .89 for Math Value, .94 for Math Self-Efficacy, and .77 for Math Ability Versus Effort. A separate score was computed for each scale to represent the average level of endorsement for each type of belief.

Child assessments

At the end of kindergarten, each child took part in an individual testing session that lasted approximately 30 min. The assessments included four tasks administered in a fixed order: Raven's test, number identification, numerical magnitude comparison, and arithmetic. The three numerical knowledge measures were based on tasks used in previous research: number identification (Göbel, Watson, Lervåg, & Hulme, 2014; Lembke & Foegen, 2009), numerical magnitude comparison (dyads, Linsen, Verschaffel, Reynvoet, & De Smedt, 2015; triads, Laski & Siegler, 2007), and arithmetic (van Galen & Reitsma, 2010). For the purposes of the present study, the original measures were adapted, as described in the following paragraphs.

In the *number identification* task, the stimuli included 6 single-digit numbers, presented in the first block of trials, followed by 24 double-digit numbers. Single-digit stimuli included unique randomly selected numbers from 1 to 9. Double-digit stimuli included unique randomly selected numbers from 10 to 99, with a constraint that there should be at least two numbers from each decade. Within each block, items were administered in a preselected random order. The child received a booklet with 1 numeral printed on each page. The tester pointed to each numeral, prompted the child to read it, and recorded the response verbatim. The response was coded as correct if the child named the digit(s) accurately (with proper verbal markers of their place value) and in the right order. If the child did not respond within 5 s, the tester provided another prompt; if there was still no response within 10 s, the tester proceeded to the next trial. After children's responses were coded, it was observed that all participants identified all single-digit numbers correctly. Thus, accuracy scores were computed as the percent correct out of 24 double-digit trials.

To examine the internal consistency of the double-digit number identification task, we computed the Kuder-Richardson statistics (KR-20), which is a version of Cronbach's alpha that is used for dichotomous responses (correct/incorrect). Kuder-Richardson statistics values greater than .70 are considered good indicators of reliability for test instruments with fewer than 50 items (Salkind, 2010). Our analysis showed that KR-20 was equal to .79, indicating good reliability of the scale.

In the *numerical magnitude comparison* task, children received a booklet with two types of items: 18 number dyads and 18 number triads (see Appendix B for sample items). Half the items of each type included single-digit numbers, and half included double-digit numbers.

Each *dyad item* was depicted within a rectangular frame divided into two squares: A different numeral was printed in each square. The task was to cross out the bigger number in a given pair. In selecting specific stimuli for the dyad task, we controlled for numeric distance and digit compatibility, following Linsen et al. (2015). Specifically, the single-digit dyads included numbers ranging from 1 to 9 with numeric distance less than five (e.g., 7 and 9). The double-digit dyads included numbers ranging from 10 to 99, such that the digits within each pair of numbers were incompatible—that is, the comparisons for decades and units led to different decisions (e.g., 78 and 93, where $7 < 9$, but $8 > 3$).

Each *triad item* was depicted within a circular frame, with the target number printed at the top of the circle and the two answer choices printed at the bottom. The task was to cross out the number at the bottom that was closest in magnitude to the target. We controlled the numeric distance so that for single-digit triads, the distance between each choice and the target was less than five (e.g., target = 5, choices = 2 and 7); and for double-digit triads, the decade distance between each choice and the target was less than five (e.g., target = 42, choices = 34 and 71). In both single- and double-digit triads, one of the choices was smaller in magnitude than the target and the other one was bigger.

In both dyad and triad tasks, children were given a demonstration trial completed by the tester, followed by 2 practice trials completed by the child with feedback and 18 test trials with no feedback. The dyad task preceded the triad task, and within each task, single-digit items were presented as a first block, followed by double-digit items. No time limit was imposed for either task. We computed the reliability statistics for the numerical magnitude comparison task as a whole, and given the results (KR-20 = .76, indicating good internal consistency), we computed a single accuracy score for the whole task as the percent correct out of 36 trials.

The *arithmetic* task included 12 addition problems, each printed on a separate sheet. All problems included single-digit addends with sums up to 10. Half the problems were in a standard format: The missing element was the sum on the right side of the equal sign (5 + 2 = __). The other half were in a nonstandard format: The missing element was one of the addends on the left side of the equal sign (3+__ = 8). On each trial, children provided a verbal response recorded by the tester. The task showed good reliability (KR-20 = .89). The score was computed as the percent correct out of 12 trials.

The version of the Raven's test selected for the present study was the Colored Progressive Matrices, designed to estimate nonverbal intelligence in children aged 5 through 11 years old (Raven, Raven, & Court, 1998). The *Raven's* task included 36 items. Each child received a booklet with 36 items, printed 1 per page. On each page, a geometric pattern with a missing piece was depicted at the top, with six answer choices depicted below. The task was to select the answer choice that would fill in the missing piece. Two practice trials were administered with feedback provided. On test trials, the child pointed to one of the choices and the tester recorded the answer. The score was calculated as the percentage of correct responses out of 36 items.

Results

First, we computed descriptive statistics and correlations between the variables to examine the pattern of relations among potential predictors (parental beliefs), mediators (home activities), and outcomes (child skills). Next, we conducted a mediation analysis using the structural equation modeling algorithm, which allowed us to simultaneously estimate multiple paths leading to each outcome.

Descriptive results

Appendix A contains descriptive statistics by item for each parent–child activity. All individual activities displayed a good range of variability, and none of them showed ceiling or floor effects. In a subsequent analysis, we used composite scores reflecting the average frequency of three types of parental input: formal math, informal math, and general enrichment activities. Table 1 presents descriptive statistics for these composite scores, along with the measures of parental beliefs and child outcomes. Table 2 shows bivariate correlations among study variables.

For all the measures included in Table 1, we examined the distribution of scores by computing the skewness coefficient and determining its significance (George & Mallery, 2010). The results showed that none of the coefficients were statistically significant, indicating that the distribution of scores for all the measures was sufficiently symmetrical to proceed with the parametric analysis. Further, all study measures exhibited substantial variability, with the exception of one belief scale. Namely, the participating parents tended to strongly endorse the view that success in math is primarily driven by effort. Perhaps, due to a lack of variability, this measure, unlike other parental beliefs, was not correlated

Table 1. Descriptive statistics.

Variable	Mean	Stand. Deviation	Range	Skewness
Child outcomes (percent correct)				
Number identification	72	28	11–100	−.78
Numerical magnitude	69	34	29–100	−.45
Arithmetic	57	23	17–100	.18
Raven's	55	18	27–83	.09
Home activities (on a scale of 1–6)				
Formal math	3.1	1.1	1.2–4.6	.17
Informal math	2.9	0.9	1.6–3.9	.45
General enrichment	3.5	1.6	2.1–4.5	.37
Parental beliefs (on a scale of 1–6)				
Math ability versus effort	5.6	0.4	5.3–5.9	−.43
Math self-efficacy	3.4	1.1	1.9–5.5	.26
Math value	3.9	2.1	2.1–5.0	−.57
Importance of school preparation	4.5	1.1	2.9–5.2	−.96

Note. In the parental belief measure "math ability versus effort," higher scores indicated a greater belief in the role of effort in math success.

Table 2. Correlations among study variables.

	Child outcomes				Home activities			Parental beliefs			
	1	2	3	4	5	6	7	8	9	10	11
Child outcomes											
1. Number identification	1										
2. Numerical magnitude	.25*	1									
3. Arithmetic	.31*	.29*	1								
4. Raven's	.18	.23*	.24*	1							
Home activities											
5. Formal math	.41**	.13	.34**	.08	1						
6. Informal math	.18	.50**	.40**	.15	−.04	1					
7. General enrichment	.22	.19	.23*	.11	.22	.10	1				
Parental beliefs											
8. Effort versus ability	.05	.09	.11	.07	.10	.06	.08	1			
9. Math self-efficacy	.03	.41**	.30*	.15	.14	.49**	.10	.09	1		
10. Math value	.23*	.35**	.31*	.09	.29**	.48**	.09	.07	.39**	1	
11. Importance of school preparation	.28*	.10	.26*	.06	.55**	.05	.21	.10	.12	.16	1

with either parent–child activities or child outcomes (see Table 2), and as such, it was not included in the subsequent path analysis.

The correlational findings revealed distinct patterns of relationships among specific beliefs, activities, and math outcomes. For example, children's scores on the *number identification* task were correlated with formal math activities and school readiness beliefs; yet they were not correlated with informal math activities and math self-efficacy. The scores on the *numerical magnitude comparison* task showed a reverse pattern. At the same time, *arithmetic* scores were correlated with both types of activities and beliefs. These distinct patterns of bivariate correlations raised the possibility of different mediation paths leading to each outcome, which was tested in the next step of analysis.

Path analysis

This analysis assessed the direct and indirect paths between parental variables and child outcomes. For each math outcome—number identification, numerical magnitude comparison, and arithmetic—we tested a path model that included three types of beliefs (school readiness, math value, and math self-efficacy) and two types of math activities (formal and informal). To determine the specificity of the relationship between home math activities and children's symbolic numeric knowledge, two covariates were included in each model: The frequency of general enrichment activities was added as a predictor of math activities to adjust for the overall quality of parenting, whereas children's Raven's scores were added as a predictor of the child math outcomes to adjust for general intelligence. The resulting models for the three outcome measures are depicted in Figures 1a, 1b, and 1c. Next, we describe each of these models and provide the model fit, indirect effects, and effect sizes.

We evaluated the *goodness of fit* for each model based on three fit indices: chi-square ratio (χ^2/*df*), comparative fit index (CFI), and root mean square error of approximation (RMSEA). A good model fit is typically indicated by a chi-square ratio of 1 to 3, CFI greater than .90, and RMSEA less than .05 (McDonald & Ho, 2002). After establishing the model fit, we computed the *indirect effect* for each mediation path as a product of standardized coefficients *ab*, where *a* is the direct effect from the distal predictor to the mediator and *b* is the direct effect from the mediator to the outcome. To establish significance of the indirect effects, we computed bias-corrected bootstrap confidence intervals; the indirect effect was considered significant if the 95% confidence interval did not contain 0 (Preacher & Hayes, 2008). Finally, for each indirect effect *ab*, we computed *effect size* as the ratio (*ab*) / (*ab* + *c*'), where *c*' is the remaining direct effect of the distal predictor on the outcome (Preacher & Kelly, 2011). This measure of indirect effect size showed what

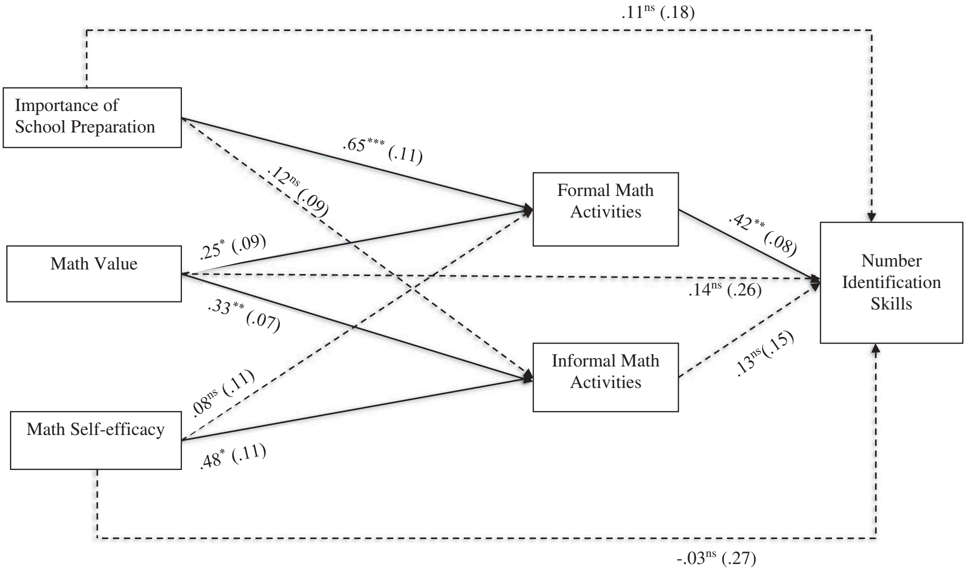

Figure 1a. Formal math activities mediated the relationship between parental beliefs and number identification skills.

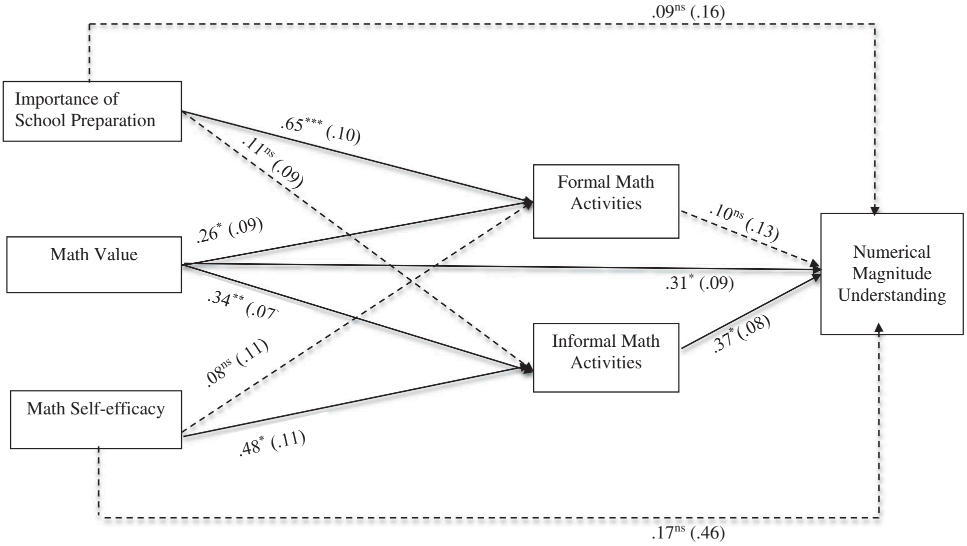

Figure 1b. Informal math activities mediated the relationship between parental beliefs and numerical magnitude understanding.

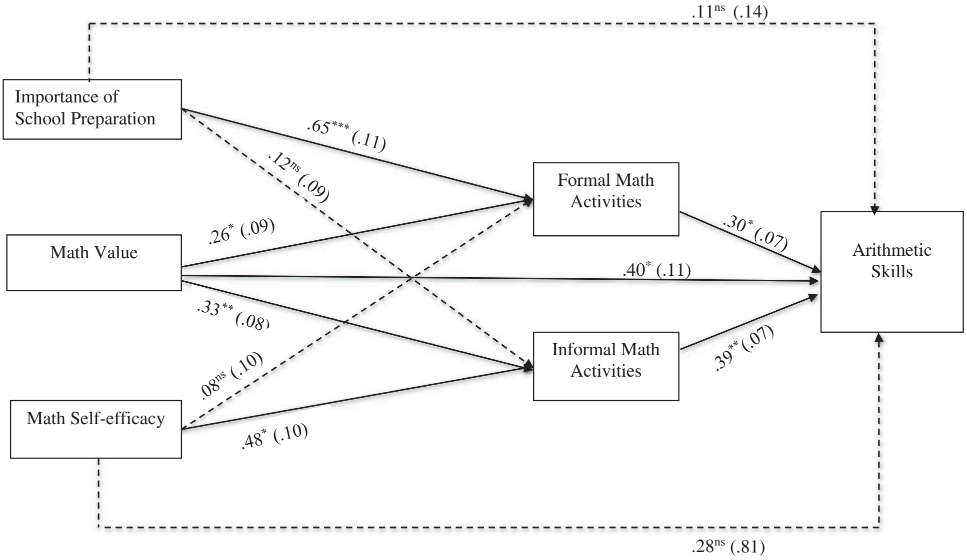

Figure 1c. Formal and informal math activities mediated the relationship between parental beliefs and arithmetic skills.

portion of the total association between the predictor and the outcome was accounted for by the mediator.

For the *number identification* skills, the path model (Figure 1a) showed a good fit with the data: $\chi^2/df = 2.2$, CFI = .97, RMSEA = .04. The frequency of *formal math activities* mediated the relationship between two types of parental beliefs—importance of school readiness and math value—and children's number identification skills. For the *school readiness belief*, the indirect effect (via formal math activities) was .27 (95% CI [0.11, 0.47]). The effect size was .49, indicating that formal math activities accounted for almost half the association between this type of belief and children's number identification skills. For *math value*, the indirect effect (via formal math activities) was .11 (95% CI [0.007, 0.38]). The effect size was .32, indicating that formal math activities accounted for almost a third of the association between this type of belief and children's number identification skills. Once the formal math activities were accounted for, the relationship between each of the two beliefs and child outcomes became nonsignificant. For all other indirect effects tested in this model, the confidence interval contained 0, indicating that these effects were not significant.

For the *numerical magnitude comparison* skills, the path model (Figure 1b) showed a good fit with the data: $\chi^2/df = 2.9$, CFI = .93, RMSEA = .05. The frequency of *informal math activities* mediated the relationship between two types of parental beliefs—math self-efficacy and math value—and children's numerical magnitude skills. For parent *math self-efficacy*, the indirect effect (via informal activities) was .18 (95% CI [0.09, 0.38]; effect size = .41). Once the informal math activities were accounted for, the relationship between this belief and numerical magnitude skills became nonsignificant. For *math value*, the indirect effect (via informal activities) was .13 (95% CI [0.04, 0.29]; effect size = .31). Thus, the informal activities accounted for about a third of the relationship between parental math value and children's numerical magnitude skills. The mediation was partial as the relation between this type of belief and child outcomes remained significant after the activities were included in the model. For all other indirect effects, confidence intervals contained 0, indicating that these effects were not significant.

For the *arithmetic* skills, the path model (Figure 1c) showed a good fit with the data: $\chi^2/df = 2.7$, CFI = .94, RMSEA = .04. This model included significant mediation paths through both formal and informal math activities. Parental beliefs in the importance of academic preparation were related to kindergartners' arithmetic performance via formal math activities (indirect effect = .19, 95% CI [0.04, 0.31]; effect size = .43). Parents' math self-efficacy was related to arithmetic skills via informal math activities (indirect effect = .18, 95% CI [0.11, 0.24], effect size = .39). In both cases, the mediation was complete such that the relation between these two beliefs and child outcomes became nonsignificant once activities were accounted for. Finally, math value was related to arithmetic skills both directly and through informal math activities (indirect effect = .13, 95% CI [0.02, 0.20]; effect size = .24). Even though the direct paths a) from math value to formal math activities and b) from formal math activities to arithmetic skills were significant, the confidence interval for the corresponding indirect path from math value to arithmetic skills contained 0 (95% CI [−0.05, 0.34]), indicating that it was not statistically significant. All other indirect paths tested in this model were also nonsignificant.

Discussion

The present study was motivated by research indicating: 1) the importance of early symbolic number skills for long-term math achievement, 2) the role of parent–child interactions as mediators of children's acquisition of number knowledge, and 3) the role of parental beliefs in shaping these interactions. Based on an analysis of symbolic number knowledge, we hypothesized specific relationships between math activities in which parents engaged children during preschool and symbolic number knowledge of the same children at the end of kindergarten. We now discuss the relevant findings and their implications for parenting and educational practice.

Relationships between home activities and child's symbolic number skills

As expected, the results indicated specific relationships of formal and informal math practices to children's math skills. The frequency of engaging preschoolers in formal math practice at home predicted accuracy on the number identification task a year later in kindergarten, whereas the same type of practice showed no relation to kindergartners' understanding of numerical magnitude. In contrast, informal math activities predicted children's understanding of numerical magnitude and not their number identification knowledge. Critically, this pattern held while controlling for general enrichment activities provided by parents and for children's intelligence.

The current findings, highlighting the unique links between parental activities and child outcomes, are consistent with the results of several previous investigations that have focused on different aspects of parental input (e.g., Gunderson & Levine, 2011; Huttenlocher et al., 2010). The overall pattern of findings across these studies suggests that specificity of the effects of parental input is a general developmental principle. For example, one study examined number talk at home in relation to a critical aspect of early math learning: cardinal-number knowledge (Gunderson & Levine, 2011). The results showed that parents' talk involving counting and labeling sets of visible objects was related to children's later cardinality knowledge, whereas other types of parental input were not. Thus, as in the present study, not all kinds of math input equally promoted this numerical skill. Rather, the findings across studies have indicated that establishing the predictors of specific skills requires a careful investigation based on theoretical predictions about the types of input involved in the development of these skills.

The pattern of results obtained in the present study was consistent with our theoretical predictions. In particular, the observed relation between formal math activities and children's number identification skills was predicted because most of these activities focused on "technical" aspects of number knowledge (e.g., recognizing numeric symbols or memorizing number facts). The relationship between informal math activities and numerical magnitude understanding was predicted because most of these daily activities provided an opportunity to think about numbers in terms of corresponding magnitudes. The latter finding adds to a growing body of literature emphasizing the learning potential of simple daily activities, such as setting a table for a family meal, shopping, or preparing food together (Mix, 2002; Vandermaas-Peeler, Boomgarden, Finn, & Pittard, 2012). Further, the present results suggest that both kinds of experiences—formal learning activities and contextualized informal activities—are valuable and complementary in that each predicts different aspects of math development.

Indicative of their complementary roles, both formal and informal activities contributed to children's ability to solve arithmetic problems. Although multiple studies have examined cognitive factors related to arithmetic development (e.g., Berg, 2008; Göbel et al., 2014), very few studies have focused on the social interaction processes that contribute to it. Previous work has shown that each component of symbolic number knowledge—numeral identification and numerical magnitude understanding—contributes to the development of arithmetic skills (Gersten, Jordan, & Flojo, 2005; Göbel et al., 2014). The present study, indicating that formal and informal activities jointly influence arithmetic knowledge, suggests that social interactions are one mechanism by which the components of symbolic number knowledge are integrated for the development of more complex mathematical skills.

Relationships between parental beliefs and home activities

Knowing which kinds of activities influence particular math outcomes is critical for designing programs that increase parents' knowledge about what they can do to facilitate children's development. However, the effectiveness of such programs may depend on parental beliefs. For instance, the likelihood of parents engaging children in math activities after being made aware of these activities may depend on whether the parents view academic preparation as an important goal for preschoolers and whether they perceive math as a valuable domain.

Indeed, we found that these sorts of beliefs predicted the frequency of math-related practices and, importantly, that the pattern of relations varied depending on the type of practice. In particular, parents' *beliefs about the importance of school preparation* were strongly related to the frequency of formal, but not informal, math practices. This finding suggests that parents tend to associate the notion of preparing children for school with decontextualized learning activities, rather than with embedding learning into daily contexts. Thus, even parents who appreciate the importance of preparing children for school may need support to recognize the wide range of activities through which this goal can be accomplished.

Another belief that was differentially related to formal and informal activities was *math self-efficacy*; however, the pattern was reversed: This belief predicted the frequency of informal, but not formal, activities. The link between math self-efficacy and informal activities may be due to parents with high efficacy being more attuned to math-related features of the environment and thus being more likely to capitalize on these features in daily interactions compared with parents with low math efficacy. The lack of relationship between math self-efficacy and formal activities may seem somewhat surprising. Although we have no definitive explanation for this finding, we can suggest a possible reason. It is conceivable that for parents with low math self-efficacy, their own perceived struggles with math actually motivate them to help their children develop better math skills. To do so, they may be more likely to employ school-like activities, which are easily associated with math learning, than daily interactions where connections to math are not obvious. This may minimize differences between parents with low and high math self-efficacy with respect to the frequency of formal activities.

Finally, the frequency of math activities was associated with parents' *math value*. In fact, unlike the previous two beliefs, math value was related to both formal and informal activities. Whereas studies of parenting typically conflate math self-efficacy and value within a single measure (Huntsinger et al., 1997; Skwarchuk et al., 2014), the present findings highlight the dissociation between these two constructs, which has been shown in motivational research (Wigfield & Eccles, 2000). This dissociation raises a possibility that even if parents are not confident in their own math skills, interventions designed to communicate the value of math for young children might positively influence their math-related interactions with children.

In sum, the frequency of formal math practices was predicted by the belief about the importance of school preparation and math value, whereas the frequency of informal math practices was predicted by math self-efficacy and math value. In other words, parents who are concerned about the academic preparation of their children and who value math skills are more likely to organize math-focused learning activities at home. On the other hand, parents with both high math value and self-efficacy may be more "into math" than other parents, leading them to incorporate (purposefully or spontaneously) math components even in those activities that are not focused on math.

Relationship among parental beliefs, home activities, and child math skills

The evidence indicating that parent–child activities are related to both parental beliefs and child outcomes raised the possibility of a mediational chain whereby activities mediate the link between parent beliefs and child outcomes. Many studies have tested various segments of this chain, but not the entire mediational model (e.g., Anders et al., 2012; LeFevre et al., 2009; Muenks et al., 2015). The few studies that have done so (e.g., Huntsinger et al., 1997) used general math assessments, such as Test of Early Mathematics Ability, for their outcome measures. Thus, the present study is unique in that it combined specificity of the analysis of child outcomes with an integration of distal and proximal parental predictors within a single mediation model.

Our approach allowed us to identify both distinct and common pathways to specific math outcomes through formal and informal math activities. In terms of distinct paths, the formal practices mediated the relationship between parental beliefs and children's number identification skills, whereas the informal practices mediated the relationship between parental beliefs and children's numerical magnitude comparison skills. At the same time, both types of practices served as parallel mediators of the relation between beliefs and arithmetic skills.

In all these models, home math practices explained a substantial proportion of the relationship between parental beliefs and child outcomes. In fact, after accounting for math activities, the relationship between parental beliefs and child outcomes typically became insignificant, with one exception—math value belief. This belief was not only linked to children's skills indirectly through home activities, but it also directly predicted two child outcomes: numeric magnitude comparison and arithmetic skills. This finding suggests that parents who strongly believe in the value of mathematical knowledge and its importance for future success may convey their belief both indirectly—through a high frequency of math-related interactions—and directly (e.g., through explicit encouragement to help children acquire more advanced numerical skills).

Understanding both direct and indirect paths between parental views, home activities, and math outcomes provides insight into potential mechanisms through which parents influence children's math development. Although the present work was correlational, it is worth noting that the relationships were consistent with theoretical hypotheses. The specificity of the associations between math-related activities and math skills examined helps minimize potential concerns that our results are due to a more general relation between the quality of the home environment and child development. Further, the longitudinal nature of the study, whereby the features of earlier parent–child interactions served as predictors of children's later skills, reduces concerns arising from the simultaneous measurement of parent input and child outcomes. Thus, the present study serves as an important intermediary between general correlational research and experimental tests of causality. The findings from this type of investigation should help guide intervention work targeting parental beliefs and activities relevant to math skills.

Limitations and future directions

Certain limitations of the present design are worth noting when interpreting the findings. First, we relied on parents' reports of math-related activities. Although previous studies have shown that parental reports provide a reliable measure of parent–child interactions (Fenson et al., 1994; Saxe et al., 1987), the self-report methodology can certainly introduce a measurement error. Second, the measures we used captured the quantitative aspects of home activities. Yet the qualitative aspects, such as the nature of parents' talk during math activities, also have been shown to play a role in the growth of math skills (Levine et al., 2010). Thus, future studies that consider the quantity as well as quality of activities might allow for an even more complete specification of the model of math development in the home context.

Third, we need to acknowledge the challenge of making a precise distinction between formal and informal categories, given the variety of ways in which parents can support preschoolers' math development. Our parental survey clearly could not include all possible combinations of math content and context in which it might be embedded. Rather, this survey included the types of activities that, based on previous work and our preliminary interviews, were more likely to occur in either a formal math learning context or informal daily context. In other words, our instrument reflected a natural confounding between the content and context of home math activities. This approach allowed us to distinguish between what generally happens in one context versus another and to examine how these general tendencies predict children's outcomes. Yet in future work, this investigation can be extended to include activities that target the same math skill (e.g., numeral writing) but vary in context (formal vs. informal) to more precisely determine the differential roles of formal and informal practices.

In addition to these limitations, we anticipate a potential concern about the generalizability of the study, given that it was conducted in Russia. Indeed, it is possible that the absolute levels of particular activities or beliefs vary cross-culturally. Yet, both the present study and previous work have shown that the types of investments that Russian parents make in their preschool children—in terms of home resources and activities—are similar to those documented in Western countries (Park, 2008; Tudge et al., 1999). Further, a recent study focusing on literacy skills of Russian children at school entry (Vasilyeva et al., 2017) replicated the model of the relationship between parental factors and child outcomes originally developed based on U.S. data (Yeung, Linver, & Brooks-Gunn, 2002). Thus, we expect that the relationships among parental beliefs, home activities, and child math skills uncovered in the present study are likely to reflect general, rather than culture-specific, patterns. Yet it would be beneficial to address this issue empirically in cross-cultural research, which may allow us to identify potential moderators of these relationships. Combining the specificity of the analysis employed in the present study with the cross-cultural perspective will help delineate unique and common pathways linking parent–child interactions to particular aspects of child development, which is critical for the refinement of constructivist theory.

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Appendix A

Parental Questionnaire: Parent–Child Activities

Parents were given a prompt: "In the last 6 months, how often did you (or another adult living with your child) engage in the following activities TOGETHER with your child?"

The prompt was followed by 18 items: *formal math activities* (Items 1–6), *informal math activities* (Items 7–12), and *general enrichment activities* (Items 13–18).

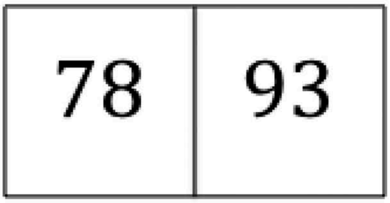
The items are listed, along with the response choices. The last two columns provide descriptive statistics for each item.

	Very rarely (1)	One to three times per month (2)	Once a Week (3)	Two to three times per week (4)	Once a day (5)	More than once a day (6)	Mean	Range
1. Practiced writing numerals	☐	☐	☐	☐	☐	☐	3.6	1–5
2. Studied how to recognize and name written numbers	☐	☐	☐	☐	☐	☐	3.9	2–5
3. Had the child work on math skills using a purchased workbook	☐	☐	☐	☐	☐	☐	3.3	1–5
4. Rehearsed the number list (1, 2, 3, etc.) and extended it to higher numbers	☐	☐	☐	☐	☐	☐	2.9	2–6
5. Rehearsed counting in unusual ways (e.g., counting backward [10, 9, 8])	☐	☐	☐	☐	☐	☐	1.9	1–4
6. Taught the child simple math facts (e.g., 2 + 1 = 3)	☐	☐	☐	☐	☐	☐	3.0	1–4
7. Counted objects during daily activities (e.g., when setting a table or doing laundry)	☐	☐	☐	☐	☐	☐	3.8	1–6
8. Measured amounts in daily activities (e.g., when cooking)	☐	☐	☐	☐	☐	☐	2.8	2–4
9. Talked about prices or counted money when shopping	☐	☐	☐	☐	☐	☐	3.0	1–4
10. Talked about time, dates, or duration of events	☐	☐	☐	☐	☐	☐	2.6	1–6
11. Discussed numbers or quantities when encountering them in stories during book reading	☐	☐	☐	☐	☐	☐	2.2	1–5
12. Played games that involved naming numbers or counting	☐	☐	☐	☐	☐	☐	3.1	2–5
13. Did arts/crafts activities at home	☐	☐	☐	☐	☐	☐	3.1	1–5
14. Played with blocks or puzzles	☐	☐	☐	☐	☐	☐	4.2	2–6
15. Read children's books	☐	☐	☐	☐	☐	☐	4.7	3–6
16. Discussed events that occurred during the day	☐	☐	☐	☐	☐	☐	2.9	1–5
17. Visited a library, a museum, or a theater	☐	☐	☐	☐	☐	☐	2.8	2–4
18. Discussed environment/nature while taking a walk	☐	☐	☐	☐	☐	☐	3.4	2–5

Appendix B

Child Assessment: Sample Stimuli From Numerical Magnitude Task

Sample Dyad Item



Sample Triad Item